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Stochastic Modeling of Temperature: An Empirical Study with Austrian Data

Status quo of Research and Institutional Debate

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Stochastic Modeling of Temperature: an Empirical Study with Austrian data

Abstract

In this paper we analyse stochastic processes for modeling daily mean temperature. We consider a mean-reverting Ornstein Uhlenbeck process with different volatility functions. The models are fitted to daily observations in the period from January 1, 1969 to December 31, 2007 at the airport Graz (Thalerhof). We compare the temperatures simulated by the models, which differ essentially in their volatility functions, with the observed temperature in the year 2008. The results could be used as a basis for pricing temperature derivatives.

Key words: Ornstein Uhlenbeck process, mean-reverting, volatility

1 Introduction

Weather has an essential influence on many business activities. Regarding for instance energy production, weather has an enourmous impact on profits and losses, because the demand for energy (electricity, gas) is highly correlated with the temperature. Therefore, a market for trading financial contracts based on temperature events has emerged in the last decade. Most of these contracts depend on certain temperature conditions, called weather derivatives and more particularly temperature derivatives. The contracts are a new type of securities and differ from insurance contracts. If an insurance owner claims a loss, he has to prove that a loss has occurred on his insured title. Another characteristic of traditional insurance contracts is that they are not geared to cover monetary losses as a consequence of temperature variation, but rather losses as a consequence of extreme weather conditions e.g. floods and drought.

On the contrary, weather derivatives are a valuable tool for managing risk, because they tend to reduce the risk caused by temperature variations. A typical example, already mentioned above, is an energy producer, for whom warm weather during the winter or low temperature during the summer may incur significant losses in earnings. The enterprise may buy a temperature derivative to compensate the losses if the temperature is too high or too low during the according season. Clearly, the enterprise has to pay for this contract, namely the premium charged by the counterparty of the contract. If the summer is too cold or the winter too warm, a contract could cover all, or a part of the incurred losses caused of temperature variation. If the seasons are not atypical, the enterprise will only lose the premium paid for the contract. There are many different structures of weather derivatives and they often depend on the needs of the investor.

Standardized weather derivatives are traded at the Chicago Mercantile Exchange (CME)[1].

[1]http://www.cmegroup.com

The market for weather derivatives exists since 1999 and there are two different classes of standardized contracts: temperature futures and options on temperature futures. All of them are linked to one of the following indices: HDD, CDD or CAT (see [3], [10]). The indices are based on measurement locations in the USA, Canada, Europe and Asia.
Especially in Europe, where the weather market is growing, there already exists another enterprise which offers weather derivative contracts, namely Celsius Pro[2]. In addition to the regulated standardizied market, there also exists an unregulated market, the so called over-the-counter (OTC) market. Nevertheless, in Austria temperature derivative contracts are rare.
Therefore, the objective of this paper is an analysis and collection of different stochastic temperature models which are considered in different papers in the literature and to apply them to temperature data from Graz Airport (Thalerhof). In the next section we take a look at the temperature data, before we start with the description of the models. In the end we simulate the different models and compare the simulated paths with the observed temperature.

2 Weather data

In this section, we consider the data set of temperature, which we got from ZAMG (Zentralanstalt für Meteorologie und Geodynamik) for the weather station located at the airport of Graz (Thalerhof). The data set consists of daily mean temperature[3] in the period January 1, 1961 until February 16, 2009, resulting in 17579 observations. The data set includes leap days entries, which we will not consider in the later explanations.

To construct the different models we only use the data set until December 31, 2007, because from January 1, 2008 until February 16, 2009 we want to compare observed with modeled temperature. This simulation should give us information about the quality of the different models. Furthermore, there are no missing values in the data set. If we take a look at the histogram of the daily average temperature (Figure 1), it indicates, that the data are not normally distributed. The Shapiro Wilk test also rejects normality. The figure shows us a left or negative skewness and a negative kurtosis, which is confirmed in Table 1. This result is caused by the cold and warm seasons in Austria.
It will be reasonable to assume normal distribution, because the histogram of the daily temperature differences (Figure 2) suggests a certain form of normal distribution. Though, small differences in the daily mean temperature will be underestimated. Nevertheless the temperature process should follow a Brownian Motion. (see also: section 6.4).

[2] http://www.celsiuspro.com

[3] usually defined to be the average of the maximum and minimum temperature over a 24h-time horizon for the specific date

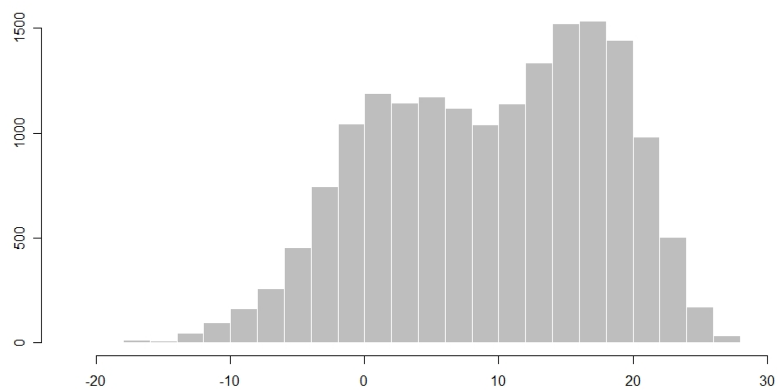

Figure 1: Histogram of daily mean temperature from Graz in the period from January 1, 1961 until December 31, 2007

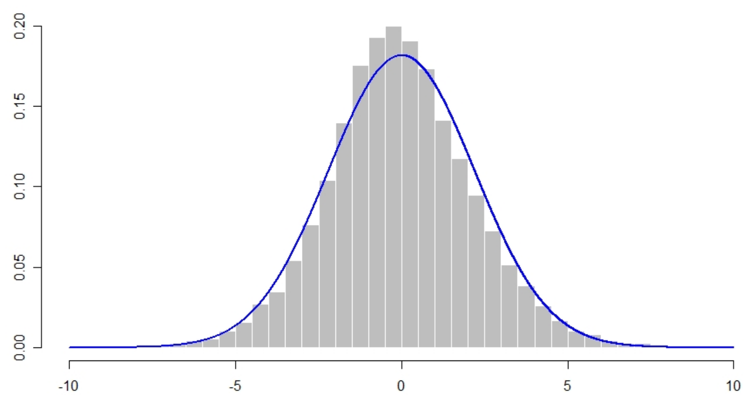

Figure 2: Histogram of the daily temperature differences from Graz in the period from January 1, 1961 until December 31, 2007

Minimum	Maximum	Mean	Skewness	Kurtosis
-20.3	28.5	9.4	-0.26	-0.81

Table 1: Summary

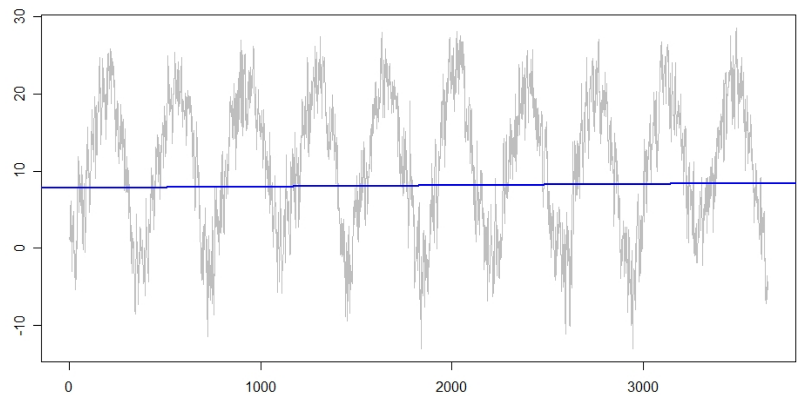

Figure 3: Observed daily mean temperature from Graz in the period from January 1, 1998 until December 31, 2007, together with the regression line.

The temperature varies between -7 degrees in the winter and about 23 degrees during the summer, as we can see in Figure 3. It is obvious that the temperature process should be a mean-reverting process with seasonality. To shape the seasonal dependence it is possible to use a trigonometric function (e.g. Sine or Cosine). A simple regression model (Table 2) shows that a weak positive, significant, linear trend exists. (in the considered period the daily mean temperature increased by about 3.18 degrees)

Intercept	Slope
7.81	0.000185

Table 2: Values of the linear regression model

There are several realistic reasons for the increase, for example global warming or the urban heating[4] or especially for us, the increase of flights at the airport.(see [11])

[4]temperature rises in areas nearby big cities

3 General assumptions for the models

Now we introduce some general assumptions, which are valid for all considered models. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$. Let us express the temperature as the solution of the following stochastic differential equation (SDE):

$$dT(t) = \theta(M - T(t))dt + \sigma(t)dB(t), \tag{1}$$

where $T(t)$ is the daily mean temperature, θ the speed of reversion (constant), $\sigma(t)$ the volatility of the process, $B(t)$ a Brownian Motion and M the mean to which the process reverts. But (1) only reverts to M (we require $E(T(t)) = M$), if the parameter is constant (see [6]). However, $M = M(t)$ should be a deterministic function for our purpose which models the trend and seasonality of the temperature. To obtain a stochastic process reverting to $M(t)$ we have to add the following term:

$$\frac{dM(t)}{dt}$$

Now we obtain a model for the evolution of temperature as an Ornstein Uhlenbeck process

$$dT(t) = \left[(\theta(M(t) - T(t)) + \frac{dM(t)}{dt}\right] dt + \sigma(t)dB(t) \tag{2}$$

whose solution is

$$T(t) = (T(0) - M(0))e^{-\theta t} + M(t) + \int_0^t e^{-\theta(t-u)}\sigma(u)dB(u). \tag{3}$$

It can be solved using the Itô-Formula [13].

4 Model A

After some general assumptions we start with the first stochastic temperature model. It is similar to the model of Alaton [1] and our analysis will be based on the same assumptions. We have to estimate the parameters of equation (2). As already mentioned above, $M(t)$ should be a deterministic function which models the trend and the seasonality. If we look at Figure 3, the behavior of the temperature suggests (according to [1]), that choosing $M(t)$ as follows

$$M(t) = a + bt + c\sin(\omega t + \phi) \tag{4}$$

with $\omega = \frac{2\pi}{365}$, gives a good fit of periodic temperature data. To estimate the numeric values of equation (4) we use ordinary least squares (OLS).

4.1 Estimation of the mean temperature function

We can write equation (4) as follows

$$M(t) = a + bt + c(\sin(\omega t)\cos(\phi)) + c(\sin(\phi)\cos(\omega t)). \tag{5}$$

It would be possible to estimate (5) using a nonlinear regression, but we transform (5) to a linear function.

$$M(t) = a + bt + \alpha_1 t_1 + \alpha_2 t_2 \tag{6}$$

with $\alpha_1 = c\cos(\phi)$, $\alpha_2 = c\sin(\phi)$, $t_1 = \sin(\omega t)$ and $t_2 = \cos(\omega t)$. To estimate the parameters of (6) we apply OLS. We get the following numerical values:

$$a = 7.86 \tag{7}$$

$$b = 0.00017 \tag{8}$$

$$c = \frac{\alpha_1}{\cos(\phi)} = \frac{-2.70}{0.24} = -10.89 \tag{9}$$

$$\phi = \tan^{-1}\left(\frac{\alpha_2}{\alpha_1}\right) = \tan^{-1}\left(\frac{-10.55}{-2.70}\right) = 1.32 \tag{10}$$

We obtain the following function for the mean temperature:

$$M(t) = 7.86 + 0.00017t - 10.89\sin\left(\frac{2\pi t}{365} + 1.32\right). \tag{11}$$

In this formula we see the weak significant trend already mentioned above. Figure 4 represents the observed mean temperature with $M(t)$.

4.2 Estimation of the volatility σ

This section utilizes ideas of Alaton [1]. To estimate σ from (2), we assume that the quadratic variation σ^2 of the temperature is nearly constant during one month, while the quadratic variation varies across the different months of the year. We assume that $\sigma(t)$ is a piecewise constant function during each month. We get 12 different values for $\sigma(t)$, $\sigma(1)$ during January, $\sigma(2)$ during February and so on.
For a specific month η ($\eta = 1, \ldots, 12$), N_η denotes the observed temperatures $T(j)$ $j = 1, \ldots, N_\eta$ during one month η. (i.e. $\eta = 1$: $N_\eta = 31 \cdot 47$ "days of January" $\cdot$ "count of years"). At first we derive one estimator for $\sigma(\eta)$ and later we estimate a second one and than we will take the average. The first estimator is based one the quadratic variation of $T(t)$ (see [2]):

$$\hat{\sigma}^2(\eta) = \frac{1}{N_\eta}\sum_{j=1}^{N_\eta - 1}(T(j+1) - T(j))^2. \tag{12}$$

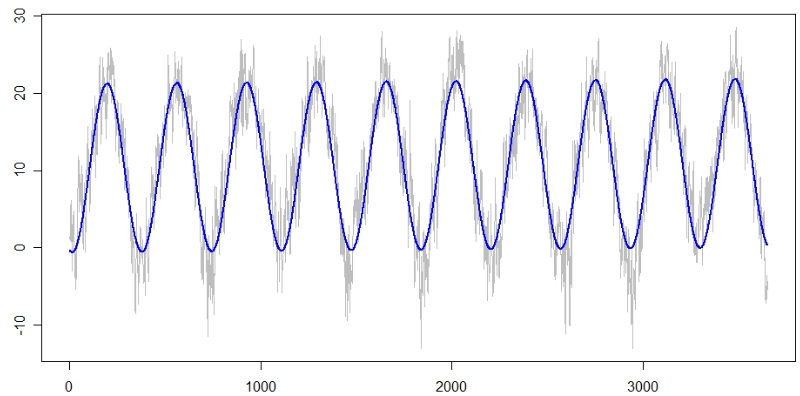

Figure 4: Observed daily mean temperature from Graz in the period from January 1, 1998 until December 31, 2007, together with mean temperature function $M(t)$.

By discretizing (2) we can derive the second estimator of $\sigma(\eta)$. The discretized equation has the following form during a given month η :

$$T(j) = M(j) - M(j-1)\theta M(j-1) + (1-\theta)T(j-1) + \sigma(\eta)\epsilon(j-1) \quad j = 1, \ldots, N_\eta \quad (13)$$

with $\{\epsilon(j)\}_{j=1}^{N_\eta}$ independent standard normally distributed random variables. We can write (13) as follows

$$\hat{T}(j) = \theta M(j-1) + (1-\theta)T(j-1) + \sigma(\eta)\epsilon(j-1), \quad (14)$$

with $\hat{T}(j) := T(j) - (M(j) - M(j-1))$. According to Brockwell [8], an efficient estimator is

$$\hat{\sigma}(\eta)^2 = \frac{1}{N_\eta - 2} \sum_{j=0}^{N_\eta} (\hat{T}(j) - \hat{\theta}M(j-1) - (1-\hat{\theta})T(j-1))^2. \quad (15)$$

To derive the second estimator of $\sigma(\eta)$, we need the estimator of θ, which is the objective of the following section.

4.3 Estimation of the mean-reverting parameter θ

According to Bibby und Sørensen [7], an unbiased estimator of θ is the zero of the equation:

$$G_n(\theta) = \sum_{i=1}^{n} \frac{\dot{b}(T(i-1);\theta)}{\sigma^2(i-1)} \left\{T(i) - E\left[T(i)|T(i-1)\right]\right\} \quad (16)$$

where n is the number of observations and $\dot{b}(T(i-1);\theta)$ denotes $\frac{\partial b}{\partial \theta}$. To solve (16) we have to determine $E\left[T(i)|T(i-1)\right]$. Equation(3), for $s \leq t$:

$$T(t) = (T(s) - M(s))e^{\theta t} + M(t) + \int_s^t e^{-\theta(t-u)}\sigma(u)dB(u), \tag{17}$$

yields

$$E\left[T(i)|T(i-1)\right] = (T(i-1) - M(i-1))e^{\theta} + M(i). \tag{18}$$

By substituting in (16) we get

$$G_n(\theta) = \sum_{i=1}^{n} \frac{M(i-1) - T(i-1)}{\sigma^2(i-1)} \Big[T(i) - (T(i-1) - M(i-1))e^{-\theta} - M(i)\Big]. \tag{19}$$

The unique solution of (19) is

$$\hat{\theta} = -\log\left(\frac{\sum_{i=1}^{n} \frac{M(i-1)-T(i-1)}{\sigma^2(i-1)}[T(i) - M(i)]}{\sum_{i=1}^{n} \frac{M(i-1)-T(i-1)}{\sigma^2(i-1)}[T(i-1) - M(i-1)]}\right) \tag{20}$$

for $i = 1, \dots, n$. $\sigma^2(i-1)$ are the associated $\sigma^2(\eta)$ estimated in (12).
Now we are able to derive the last unknown parameters of (2). The numerical value of the estimator of θ is

$$\hat{\theta} = 0.26. \tag{21}$$

The estimators of σ are listed in Table 3, and we see that the volatility of the temperature during the summer months is lower than in the winter. It is also obvious that estimator 1 and 2 are of the same magnitude for almost all months.

5 Model B

This model is a modification of Model A and was developed by Bhowan [6]. The main difference of these two models lies in the estimation of the volatility. In Model A we argue that the volatility of the temperature is nearly constant during a month, but varies across the year. This means that σ is a piecewise constant function, which changes monthly. Bowhan [6] proposes to apply a stochastic process for the volatility. The volatility changes randomly on a monthly basis, but is still constant during one month.

5.1 Estimation of the mean temperature function

To estimate the mean temperature function, we use the same idea as in Section 4.1. To mention the parameters one more time:

$$a = 7.86 \quad b = 0.00017 \quad c = -10.89 \quad \phi = 1.32.$$

Month	1st Estimation	2nd Estimation	Average
January	2.74	2.74	2.74
February	2.58	2.58	2.58
March	2.70	2.70	2.70
April	2.37	2.38	2.37
May	2.31	2.32	2.32
June	2.16	2.17	2.17
July	2.04	2.05	2.05
August	1.96	1.96	1.96
September	2.12	2.12	2.12
October	2.46	2.46	2.46
November	2.71	2.71	2.71
December	2.74	2.74	2.74

Table 3: Estimators of σ

5.2 Estimation of the volatility process σ

Before we consider a stochastic mean reverting process for the volatility let us consider Figure 5 representing the observed monthly volatility. To calculate the monthly volatility take equation 12 and replace N_η by the number of days of the according month (i.e. 31 for January 1961 and so on), which yields to 564 different volatilities, for each month the according value. The volatility should revert to a long term trend, that is why the stochastic differential equation has the form,

$$d\sigma(\tau) = -\theta_\sigma(\sigma(\tau) - M_\sigma)d\tau + \sigma_\sigma dB_\sigma(\tau). \tag{22}$$

M_σ is constant and the estimated value is

$$\hat{M}_\sigma = 2.14. \tag{23}$$

Two parameters remain to be estimated, namely σ_σ and θ_σ. In the case of σ_σ we use the estimator of the quadratic variation

$$\hat{\sigma}_\sigma^2 = \frac{1}{n}\sum_{j=0}^{n-1}(\sigma(j+1) - \sigma(j))^2 \tag{24}$$

with $\sigma(j)$ the quadratic variation of the temperature during the month j. We obtain

$$\hat{\sigma}_\sigma = 0.51. \tag{25}$$

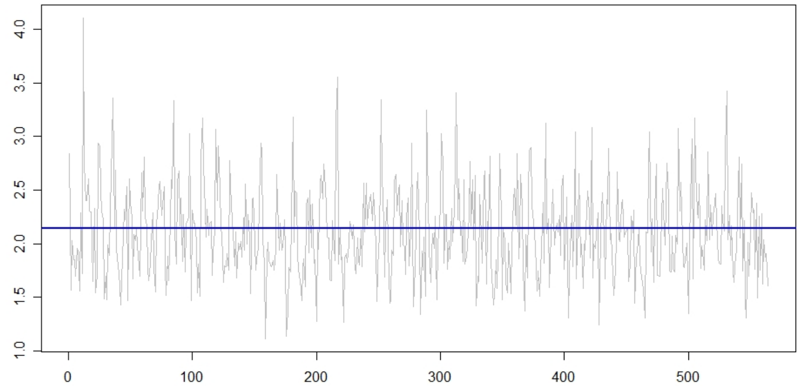

Figure 5: Observed monthly volatility of temperature from Graz in the period from January 1, 1998 until December 31, 2007, together with $\hat{M}_\sigma$.

θ_σ is estimated by a modification of (20)

$$\hat{\theta}_\sigma = -\log\left(\frac{\sum_{i=1}^{n}\frac{M_\sigma-\sigma(i-1)}{\sigma^2(i-1)}[\sigma(i)-M_\sigma]}{\sum_{i=1}^{n}\frac{M_\sigma-\sigma(i-1)}{\sigma^2(i-1)}[\sigma(i-1)-M_\sigma]}\right), \tag{26}$$

then

$$\hat{\theta}_\sigma = 1.39 \tag{27}$$

5.3 Estimation of the mean-reverting parameter θ

In this model the estimation of the mean-reverting parameter is similar to Section 4.3. Only the $\sigma^2(i-1)$ must be modified. Performing the modified calculation in (20) gives

$$\hat{\theta} = 0.21. \tag{28}$$

6 Model C

The analysis of this last model is based on the model of Benth [3]. He proposes to model the volatility as a truncated Fourier series. This choice leads to a seasonal volatility. The procedure of estimating the parameters of equation (2) completely differs from Model A and B. At first we discretize the solution of the Ornstein-Uhlenbeck process (3). This calculation yields to following term:

$$\Delta T(t) = \Delta M(t) - (1 - e^{-\theta})(T(t) - M(t)) + e^{-\theta} \int_t^{t+1} e^{-\theta(t-u)} \sigma(u) dB(u) \qquad (29)$$

with $\Delta T(t) = T(t+1) - T(t)$. Next we approximate the stochastic integral, which yields

$$\Delta T(t) = \Delta M(t) - (1 - e^{-\theta})(T(t) - M(t)) + e^{-\theta} \sigma(t) \Delta B(t). \qquad (30)$$

This equation can be written as a time series model (AR(1)-Model) of the following form:

$$\tilde{T}(t+1) = \rho \tilde{T}(t) + \tilde{\sigma}(t) \epsilon(t) \qquad (31)$$

with $\tilde{T}(t) = T(t) - M(t)$, $\tilde{\sigma}(t) = \rho\sigma(t)$, $\rho = e^{-\theta}$ and $\epsilon(t)$ i.i.d standard normally distributed. To estimate (31) we have to follow several steps. At first we have to remove the seasonality and the linear trend.

6.1 Estimation of the mean temperature function

We already mentioned that $M(t)$ should be a deterministic function modeling the trend and the seasonality. In this model we specify $M(t)$ to be of the form

$$M(t) = a + bt + b_1 + b_2 \cos\left(\frac{2\pi(t - b_3)}{365}\right). \qquad (32)$$

At the beginning we already showed that there exists a weak linear trend (see Figure 3 and Table 2). After removing the trend we can determine the seasonal part of $M(t)$. This yields to the following parameters of $M(t)$:

$$b_1 \approx 0 \qquad b_2 = -10.89 \qquad b_3 = 14.55$$

6.2 Estimation of the mean-reverting Parameter θ

Now we use the de-trended and de-seasonalized temperature series to estimate the coefficient of (31). In other words, we regress todays mean temperature against those of yesterday. The value of the mean-reverting parameter, which is significant, is

$$\rho = 0.80$$

and corresponds to

$$\hat{\theta} = -\ln(\rho) = 0.22.$$

Before we estimate the volatilty σ, we take a look at the autocorrelation function (Figure 6) of the residuals of the AR(1) model of the de-trended and de-seasonalized mean temperature. We observe high values of the autocorrelation for the first lags. For the higher lags it seems that the values are varying randomly around zero. The autocorrelation function of the sqared residuals (Figure 7) indicates the fact of time dependency. Moreover a clear seasonal variation exists.

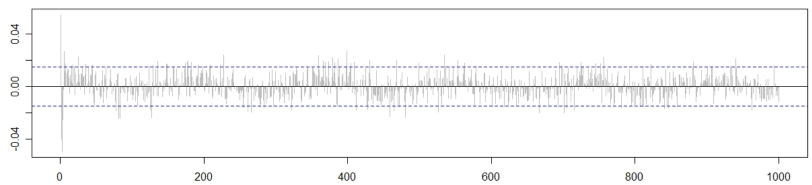

Figure 6: Autocorrelation function of the residuals of the mean temperature at the airport Graz

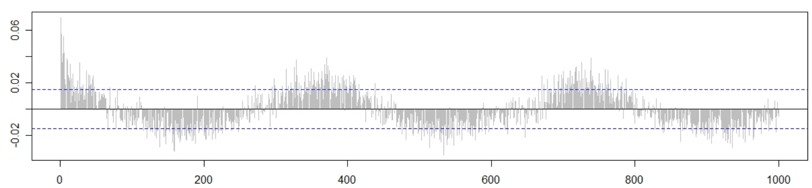

Figure 7: Autocorrelation function of the square residuals of the mean temperature at the airport Graz

6.3 Estimation of the volatility process σ

The estimation of the seasonal volatility of the residuals will be done in several steps. First, we define the form of $\sigma^2(t)$. We already mentioned above that the volatility will be modeled with a truncated Fourier series

$$\sigma^2(t) = c + \sum_{i=1}^{I} c_i \sin(2i\pi t/365) + \sum_{j=1}^{J} d_j \cos(2j\pi t/365). \tag{33}$$

At first we group the residuals of the AR(1)-model in 365 groups, that means we get 47 observations for a particular date (i.e. May 1). Taking the average of the square of each group we obtain the volatility. Next we choose (according to [3]) $I = J = 4$ in equation (33). Recall

$$\sigma^2(t) = \tilde{\sigma}^2(t)/\rho^2. \tag{34}$$

In Figure 8 we see the empirical volatility with the fitted function. The highest volatility occurs during the winter period, while fall and summer season have lower volatilities. This confirms our result of the simpler model form Section 4. In Tabel 4 we see the fitted parameters of $\sigma^2(t)$.

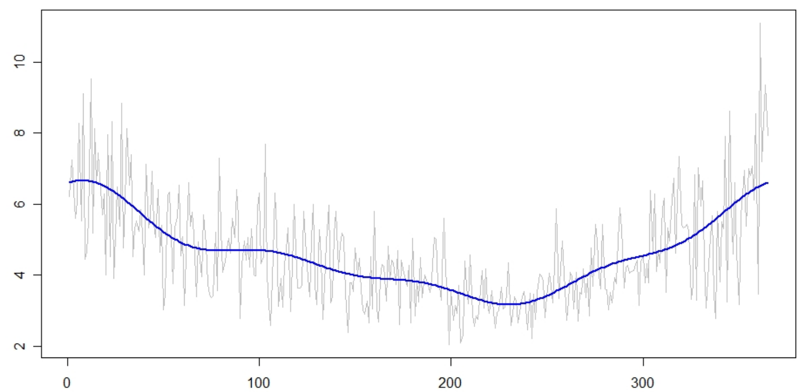

Figure 8: Empirical volatility with the fitted function $\tilde{\sigma}(t)$

c	c_1	d_1	c_2	d_2	c_3	c_3	c_4	d_4
6.74	0.74	1.95	-0.18	0.77	0.15	0.18	0.14	0.26

Table 4: Estimators of $\sigma^2(t)$

6.4 Non-normality

After removing the temporal phenomena in the volatility we obtain the autocorrelation functions, which are presented in Figure 9 and Figure 10. We can see that the seasonality in the autocorrelation function of the square residuals (Figure 10) has been removed. The autocorrelation in the first lags is still existing. This should be an indicator to use more refined models, but this would lead to a significant complication of later calculations of

futures and options prices.
Although the histogram of the residuals (Figure 11) seems to be standard normal distributed, there still exists another problem. The Shapiro Wilk test rejects normal distribution and the residuals are left skewed.
Benth and Saltyte-Benth suggest in [4] to model the residuals by generalized hyperbolic distribution. Such models are able to capture small peaks in the center, which we see in the histogram. This modification would lead to a Lévy Process. In [3] the authors mention that such processes may be hard to use for pricing derivatives and they think that the assumption of $i.i.d$ standard normal Residuals is remarkable.

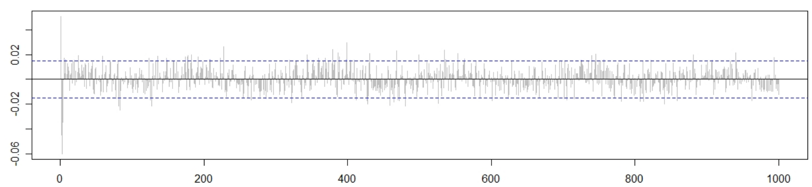

Figure 9: Autocorrelation function of the residuals of the mean temperature at the airport Graz, after dividing out the volatility function $\tilde{\sigma}$(t)

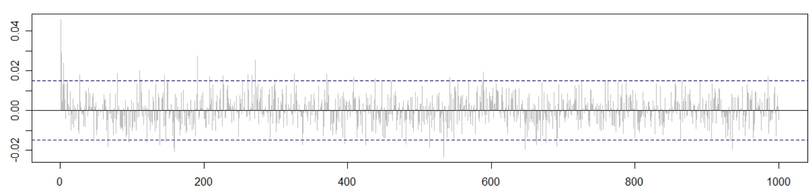

Figure 10: Autocorrelation function of the square residuals of the mean temperature at the airport Graz, after dividing out the volatility function $\tilde{\sigma}$(t)

7 Simulation

In this section we want to simulate the different models and compare the simulated values with the observed temperature in the period January 1, 2008 until Febrary 16, 2009, as already mentioned. For simulating the path we have to discretise (2) and and (22),

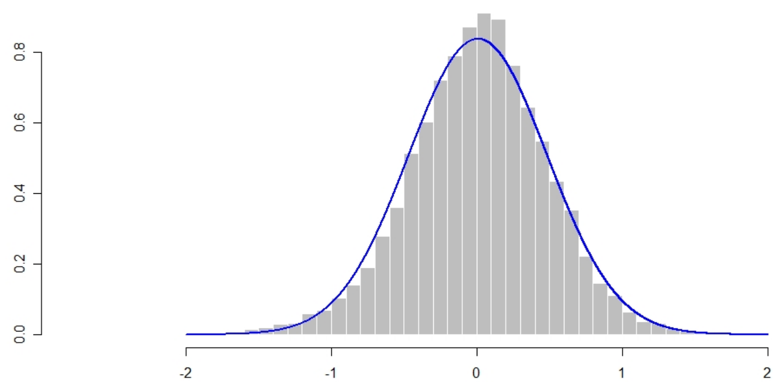

Figure 11: Histogram of the residuals of the temperature from Graz with the standard normal density

especially for Modell B. Using the Euler Scheme of approximation (see [12]) we obtain the following equations

$$T(t+1) = T(t) + \theta(M(t) - T(t)) + \frac{dM(t)}{dt} + \sigma(\tau)Y_1 \tag{35}$$

$$\sigma(\tau) = \sigma(\tau-1) + \theta_\sigma(M_\sigma - \sigma(\tau-1)) + \sigma_\sigma Y_2 \tag{36}$$

with Y_1, Y_2 $i.i.d$ standard normal distributed random variables. $\sigma(\tau)$ in (35) for Model A are given in Table 3 for the according month. In Model B the volatility will be simulated using (36) for the according month. For the last Model the volatility is given by the truncated Fourier Series (33). The other parameters and estimators are given in the previous sections.
The simulation will be done in several steps. A simulated path will be the average of 5 separately simulated paths (i.e. a simulated path of Model A is the average of 5 separately simulated paths of Model A). To limit the number of simulated paths to 5 is just empirical. This assumption makes sense, because if the number is too high the simulated temperature is too smooth and if the number is too low the variation of the path is very high. To enable the comparison of the models, the generated normal distributed random variables will be the same for the according run of each model (i.e. the random varaiables of the first run of Model A, B and C are the same).
Carrying out various simulation runs shows that the solutions of the different models are nearly the same. Furthermore, the errors between the observed and fitted values of the

different models are of similar size. Models of different complexity give us nearly the same result. It's impossible to say which model is the most suitable for simulating the observed temperature in the period from January 1, 2008 to February 16, 2009.
Figure 12 shows a simulated path of Model C and the observed values in the period from January 1, 2008 to February 16, 2009.

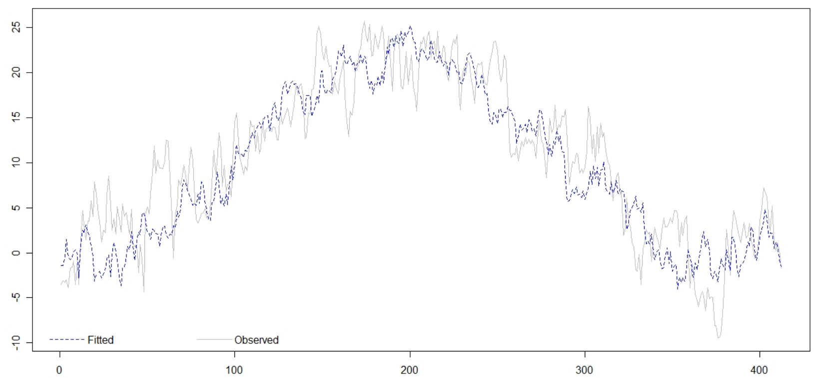

Figure 12: Simulated Path (Model C) using the Euler Scheme, together with the observed mean temperature in the period from January 1, 2008 to February 16, 2009

8 Conclusion and Further Research

In this work we collected different stochastic temperature models and applied them to observed temperature data from Graz. These models could be a basis for pricing temperature derivatives in Austria based on different indices. It should also be possible to construct a spatial-temporal model for Austria or Styria on the basis of these models. Another step could be a modification of the models to get better models for Austrian temperature data. The use of Lévy processes could be reasonable. To advance models it might be helpful to consider larger models where the temperature is only one of many variables.

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